EBG-ANTENNA ANALYSIS : UNIFICATION OF FREQUENCY AND ANGULAR DEPENDENCIES

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ABSTRACT : In this paper, we present an original method to study planar EBG based antennas. From this method, which unifies the frequency and angular dependencies of the EBG structure, we obtain an analytical expression of the directivity of the structure versus frequency. To our knowledge, these results concerning EBG structures have never been presented.

I. INTRODUCTION

Several recent works [1][2][3][4] deal with directive antennas based on EBG (Electronic Band Gap) structures. In these works the directivity characteristic is generally studied numerically. The purpose of this presentation is to establish an analytical expression and to show that the frequency and spatial behaviors of EBG structures are tightly interrelated. To this end we propose a new simple method for analyzing EBG-Antenna. Inspired from the antenna array theory [5], this method can allow, for example, to obtain the radiation pattern of an EBG-Antenna from the frequency curves. We will consider first a point source inside multiple Partially Reflecting Surfaces (PRS). Then we will study in more details the case of one PRS on each side of the point source, estimating the directivity and the performance of this EBG-Antenna. Finally, we will focus on a structure composed of a ground plane in the same plane of the source.

II. GENERAL EQUATIONS

Let us consider an ideal point source inside a periodic structure composed of 2n identical partially reflecting surfaces (Figure 1). In practice, the source can be a dipole as in ref. [2][3]. The source creates plane waves uniformly propagating in all θ directions. Let (r) and (t) be respectively the complex reflection and transmission coefficients of a single layer ($r=|r|\exp(j\varphi_r)$, $t=|t|\exp(j\varphi_t)$).

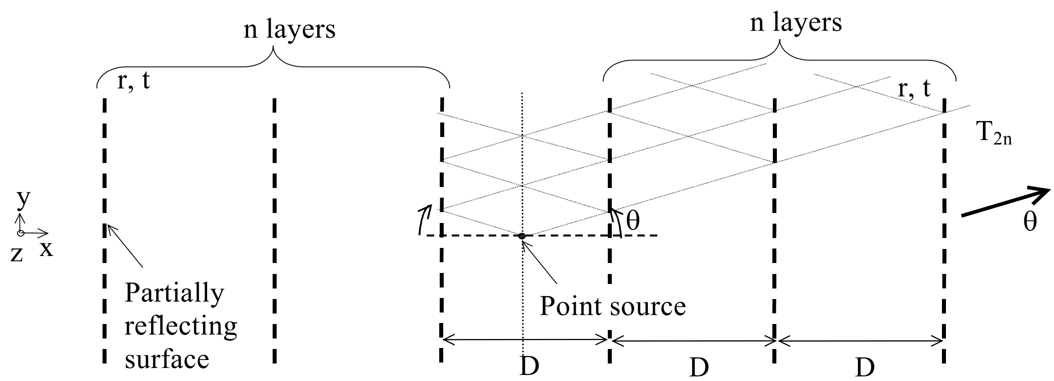

Figure 1 – Point source inside multiple (2n) Partially Reflecting Surfaces (PRS).

We have calculated the transmission coefficient $T_{2n}(\psi)$ of the structure due to two symmetrical plane waves at a given angle θ (Figure 1). We have shown [6] that the transmission coefficient $T_{2n}(\psi)$ can be written in a simple closed form :

$$T_{2n}(\psi) = \frac{t_n(\psi)\exp(-j\psi/2)}{1-r_n(\psi)\exp(-j\psi)} \tag{1}$$

t_n and r_n are evaluated by recursive equations from (r, t) [6] :

$$t_n(\psi) = \frac{t_{n-1}(\psi).t.\exp(-j\psi)}{1 - r_{n-1}(\psi).r.\exp(-j2\psi)} \tag{2}$$

$$r_n(\psi) = r_{n-1}(\psi) + \frac{t_{n-1}(\psi)^2.r.\exp(-j2\psi)}{1 - r_{n-1}(\psi).r.\exp(-j2\psi)} \tag{3}$$

$$t_1(\psi) = t \ , \qquad r_1(\psi) = r \ , \tag{4}$$

and $$\psi = kD\cos(\theta) \tag{5}$$

k is the free space wave number and D is the distance between two consecutive partially reflecting surfaces (Figure 1).

It is important to note that the expression (1) unifies the frequency and incidence angle dependencies in a unique variable ψ=kDcos(θ). Another point is that $T_{2n}(\psi)$ is a periodic function of ψ with period 2π. The knowledge of one period ($0\leq\psi\leq2\pi$) is sufficient to extract the frequency curves ($0\leq f<\infty$) and/or the radiation pattern ($-\pi/2\leq\theta\leq\pi/2$).

In Figure 2, we give an example of $|T_{2n}(\psi)|$ for |r|=0,7.exp(j 2.35), |t|=0,71.exp(j0.78) [6] (for n=1,2, 3). To simplify the study, we consider that (r, t) don't depend on frequency or on incidence angle. $T_{2n}(\psi)|$ contains two types of pass-bands : pass-bands with high picks, and pass-bands with picks limited to one. The values of $|T_{2n}(\psi)|$ higher than one are explained physically in ref. [6][7]. It does not correspond directly to antenna directivity as it was used in [8][9].

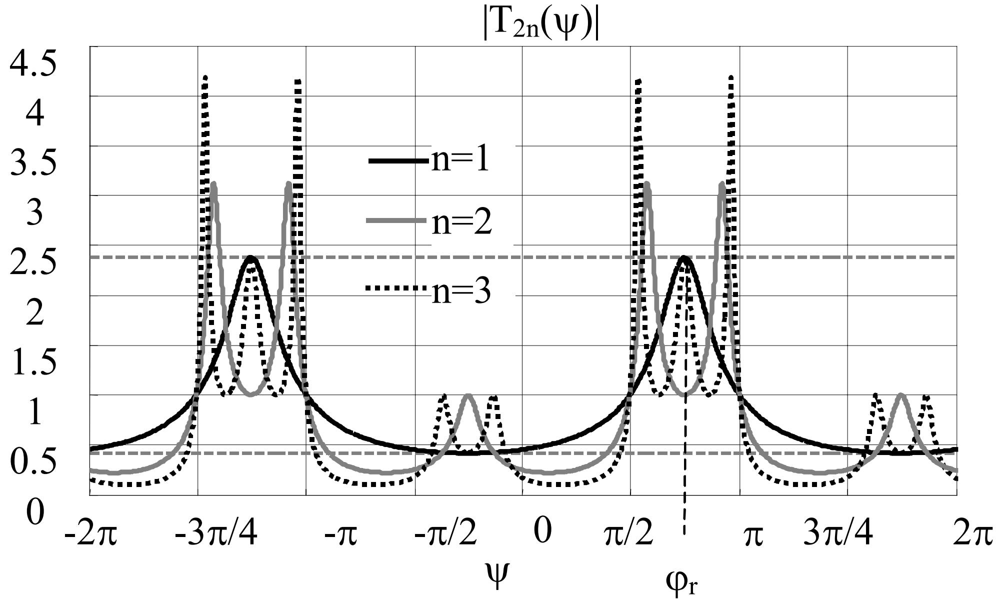

Figure 2 – Transmission coefficient versus ψ for n=1, 2, 3 (|r|=0,7.exp(j 2.35), |t|=0,71.exp(j0.78)).

Now it's easy to pass from the ψ domain to the frequency domain (for a given angle θ) or to the angular domain (for a given frequency), by using the classical method of antenna array theory [5].

III. STRUCTURE WITH ONE LAYER ON EACH SIDE OF THE SOURCE

III. 1 Extraction of frequency curve and radiation pattern

We consider one PRS on each side of the source (Figure 3). For this simple structure, we will show how to pass from the ψ domain to the frequency or angular domains. We will also evaluate the directivity of this structure and calculate the frequency which gives the maximum of directivity.

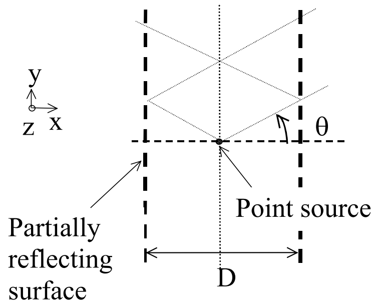

Figure 3 – Point source inside two partially reflecting surfaces

The magnitude of the transmission coefficient $T_2(\psi)=\frac{t.\exp(-j\psi/2)}{1-r.\exp(-j\psi)}=\frac{t.\exp(-j\psi/2)}{1-|r|.\exp(-j(\psi-\varphi_r))}$ is given in Figure 4. To pass from the ψ domain to the frequency domain we use the following transformation : $f=\frac{\psi.c}{2\pi D\cos(\theta)}$, which we wrote for θ=0° $f=\frac{\psi.c}{2\pi D}$. c is the speed of light. The resonance frequency $f=f_0=\frac{\varphi_r.c}{2\pi D}$ correspond to $\psi=\varphi_r$. $|T_2|$ is plotted versus the frequency f in the second abscissa of Figure 4.

To pass from the ψ domain to the angular domain we use $\theta=\pm\arccos\left(\frac{\psi}{kD}\right)$. For $f=f_0$, $\theta=\pm\arccos\left(\frac{\psi}{\varphi_r}\right)$.We use a graphical method [5] as it is shown in Figure 4. Note that the well known "visible region" in the array theory starts from the 0 frequency and finishes at the frequency considered.

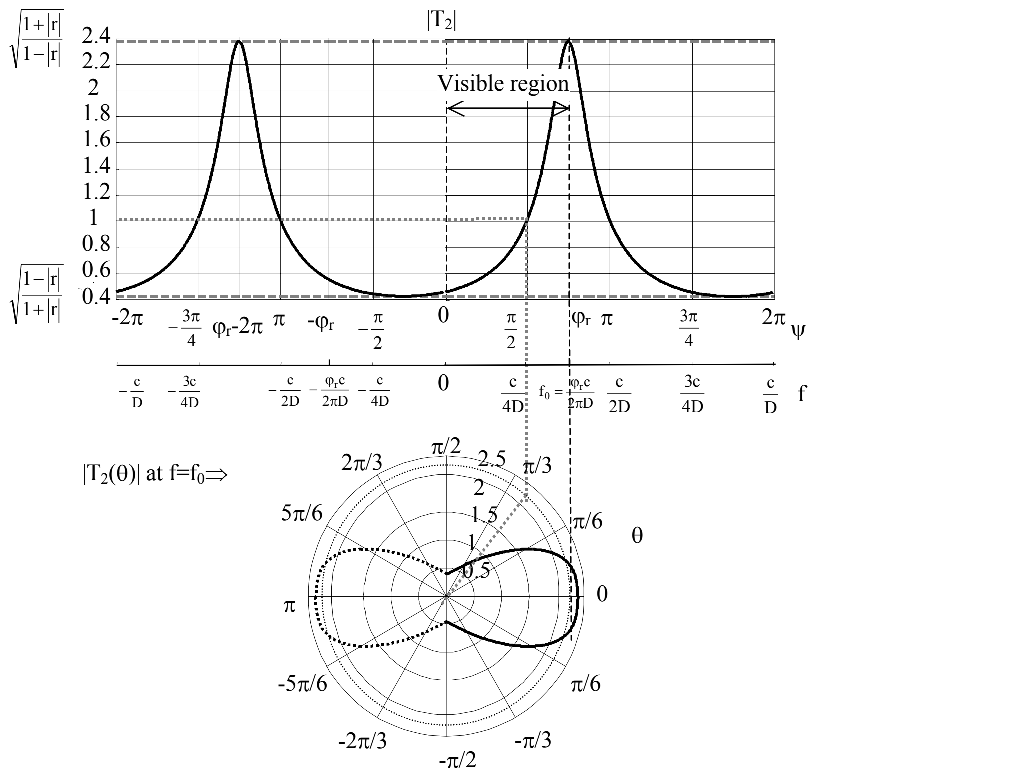

Figure 4 – $|T_2|$, transformations from the ψ domain to the frequency domain (at θ=0°) and to the angular domain (at $f=f_0$)

It's interesting to note that the lower and upper envelops of the transmission coefficient $|T_2|$ are given by [6] :

$$|T_2|_{max} = \frac{|t|}{1-|r|} = \frac{\sqrt{1-|r|^2}}{1-|r|} = \sqrt{\frac{1+|r|}{1-|r|}}, \qquad |T_2|_{min} = \frac{|t|}{1+|r|} = \frac{\sqrt{1-|r|^2}}{1+|r|} = \sqrt{\frac{1-|r|}{1+|r|}} = \frac{1}{|T_2|_{max}} \qquad (6)$$

Note that $|T_2|_{max}$ should not be considered as the gain enhancement of the structure as it is proposed in ref. [8][9]. We will see later how to calculate the directivity and then the true performance of the structure.
Figure 5 gives $|T_2(\theta)|$ for a frequency higher than f_0 : in this case the maximum is not at $\theta=0°$ and multiple lobes appear.

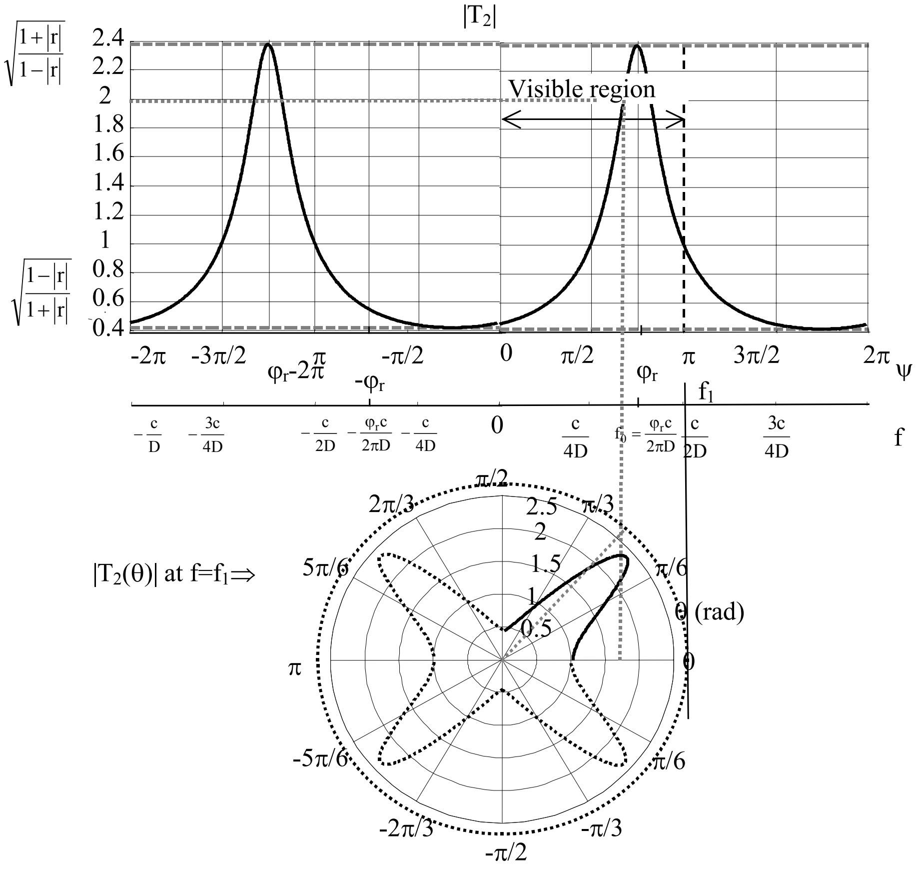

Figure 5 – $|T_2(\psi))|$ and $|T_2(\theta)|$ at $f=f_1>f_0$.

III. 2 Beam width versus frequency

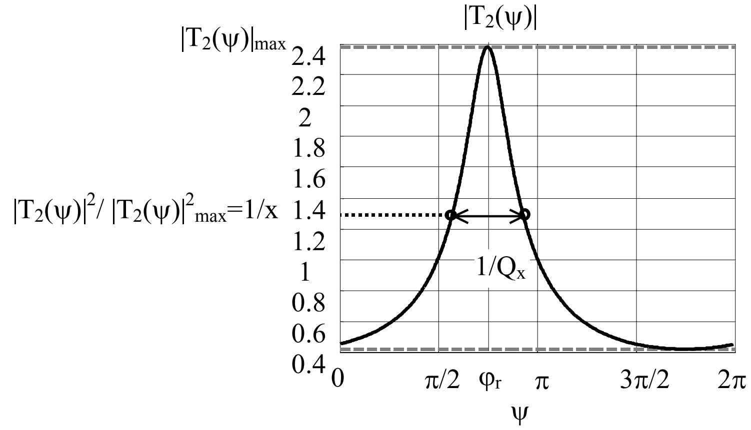

Figure 6 – Quality factor at 1/x.

We call Q the quality factor of $|T_2|$ which is obtained calculating the inverse of the bandwidth at –3dB of $|T_2|$ multiplied by f_0.
Let us also call Q_x the quality factor of T_2 obtained for a bandwidth corresponding to $|T_2|^2=|T_2|_{max}^2/x$ (then $Q_2=Q$) (Figure 6). In the Appendix, Q is calculated in function of r and Q_x is calculated in function of Q.

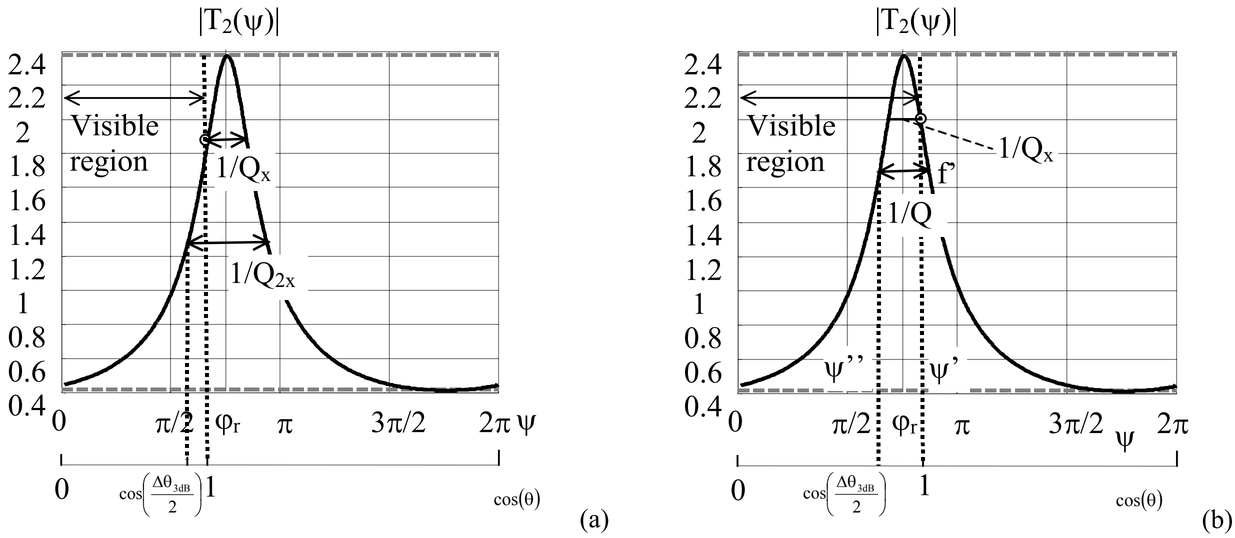

Figure 7 – Link between quality factor and beam width (a) for a frequency smaller than f_0 (b) for a frequency greater than f_0 and smaller than f'.

Now we will use the results presented in the Appendix to estimate the beam width $\Delta\theta_{3dB}$ versus frequency.
For a frequency smaller than f_0 (see Figure 7a) we have to resolve this following equation :

$$\frac{1}{Q_{2x}}-\frac{1}{Q_x}=2\left(1-\cos\left(\frac{\Delta\theta_{3dB,f\le f_0}}{2}\right)\right) \tag{7}$$

Considering that $\Delta\theta_{3dB}$ is small we can write :

$$\left(\frac{\Delta\theta_{3dB,f\le f_0}}{2}\right)^2\approx\frac{1}{Q_{2x}}-\frac{1}{Q_x} \tag{8}$$

Then (using (22)): $$\boxed{\Delta\theta_{3dB,f\le f_0}\approx 2\sqrt{\frac{\sqrt{2x-1}-\sqrt{x-1}}{Q}}} \tag{9}$$

For a frequency greater than f_0 but smaller than f' (see Figure 7b), this is written :

$$\frac{1}{Q_x}+\frac{1}{Q}=2\left(1-\cos\left(\frac{\Delta\theta_{3dB,f_0\le f\le f'}}{2}\right)\right) \tag{10}$$

Then : $$\boxed{\Delta\theta_{3dB,f_0\le f\le f'}\approx 2\sqrt{\frac{\sqrt{x-1}+1}{Q}}} \tag{11}$$

Note that we obtain easily the beam width at f_0 in function of Q, by putting x=1 in (11), we find $\Delta\theta_{3dB,f_0}\approx\frac{2}{\sqrt{Q}}$ which is in accordance with the result in ref. [3][10]. We call this relation, the relation of performance of the structure.

This study gives a more complete result than [3][10], as all frequencies are considered. Figure 8a gives the beam width $\Delta\theta_{3dB}$ versus the frequency ((9) and (11)) from 0 to f' and Figure 8b gives $\pi/\Delta\theta_{3dB}$ which we consider as an elegant evaluation of the directivity D_0 at $\theta=0°$ of the structure.

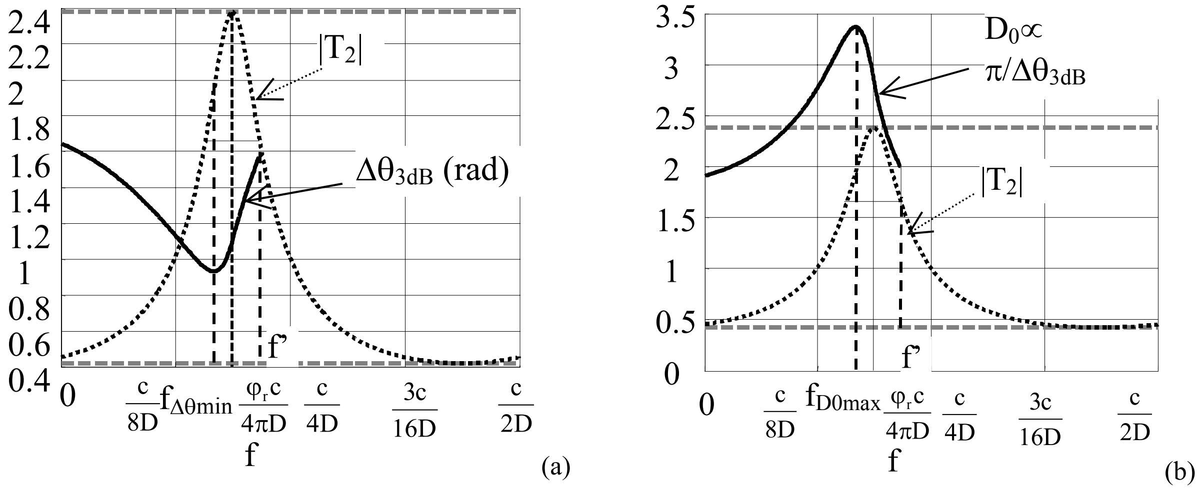

Figure 8 – (a) $\Delta\theta_{3dB}$ and (b) $\pi/\Delta\theta_{3dB}$ versus frequency.

Note that the minimum of $\Delta\theta_{3dB}$ and so the maximum of directivity is not obtained at f_0 but at a frequency slightly smaller than f_0. To obtain this frequency we calculate the value of x which gives the maximum of the following function : $g(x)=\sqrt{2x-1}-\sqrt{x-1}$ (see equation (9)).

after derivation : $g'(x)=\dfrac{2\sqrt{x-1}-\sqrt{2x-1}}{\sqrt{(2x-1)(x-1)}}$; $g'(x)=0 \Leftrightarrow x=1,5$

The minimum beam width is then : $\boxed{\Delta\theta_{3dB,min}\approx\sqrt{\dfrac{2}{Q}}}$ *(12)*

We obtain the corresponding frequency by resolving $2\dfrac{f_0-f_{\Delta\theta\,min}}{f_0}=\dfrac{1}{Q_{\Delta\theta\,min}}\approx\dfrac{\sqrt{0.5}}{Q}=\dfrac{1}{\sqrt{2}Q}$

Finally, $\boxed{f_{\Delta\theta\,min}\approx f_0\left(1-\dfrac{1}{2\sqrt{2}Q}\right)}$ *(13)*

IV. STRUCTURES WITH A GROUND PLANE

Let us consider now that a ground plane is placed in the same plane as the point source (in a manner that there is no radiation in the left side of the source) and there is only n PRS layers placed at a distance D/2 from the source. The PRS layers are spaced from each other by a distance D. We call $T_{n,gp}$ the transmission coefficient of the structure, whose expression becomes [6] :

$$T_{n,gp}(\psi)=\frac{t_n(\psi)\exp(-j\psi/2)}{1-r_n(\psi)\exp(-j(\psi+\pi))} \qquad (14)$$

Note that $|t_n(\psi)|$ and $r_n(\psi)$ are periodic with period π (see equations (2) and (3)). Therefore :

$$\left|T_{n,gp}(\psi)\right| = \frac{|t_n(\psi)|}{|1 - r_n(\psi)\exp(-j(\psi+\pi))|} = \frac{|t_n(\psi+\pi)|}{|1 - r_n(\psi+\pi)\exp(-j(\psi+\pi))|} = |T_{2n}(\psi+\pi)| \quad (15)$$

We deduce that the $|T_{n,gp}(\psi)|$ curve can be obtained from the $|T_{2n}(\psi)|$ one by π shifting it to the right as we can see in Figure 9. This is equivalent to add π to the phase of the reflection coefficient φ_r. Note that in our example φ_r is positive. We can imagine that for φ_r negative the first $|T_{n,gp}(\psi)|$ pick will be higher than one.

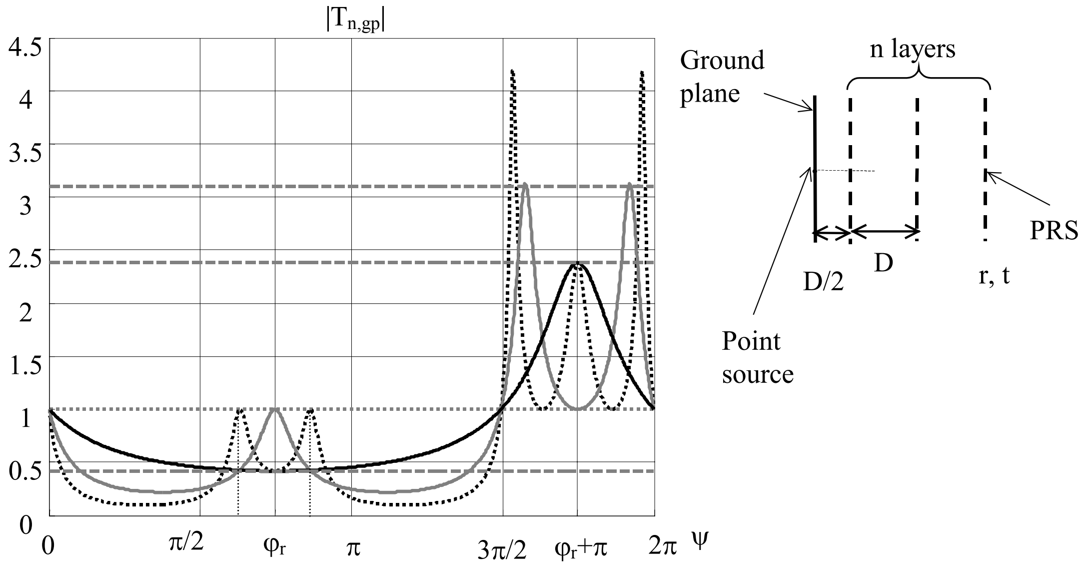

Figure 9 – Transmission coefficients $|T_{n,pm}|$, for n= 1,2 et 3,in one period [0,2π].

V. CONCLUSION

We have presented a new and effective method to analyze 1-D Electromagnetic Band Gap based antennas. Considering Partially Reflecting Surfaces having characteristics independent of frequency and angle, we have estimate the directivity versus frequency and the frequency of maximum directivity for the simple structure composed of one layer on each side of the source. We have shown that EBG structures composed of a ground plane can be analyzed using the study of structures composed of multiple layers on each side of the source.

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APPENDIX

To calculate Q we resolve the following equation :

$$\left|\frac{t}{1-r.\exp(-j\psi')}\right| = \frac{1}{\sqrt{2}}\sqrt{\frac{1+|r|}{1-|r|}} \tag{16}$$

which gives us ψ’ and ψ’’ the values of ψ corresponding to the half power (see Figure 7b) :

$$\psi' = \left(\varphi_r + \arccos\left(1-\frac{1}{2}\frac{(1-|r|)^2}{|r|}\right)\right), \qquad \psi'' = \left(\varphi_r - \arccos\left(1-\frac{1}{2}\frac{(1-|r|)^2}{|r|}\right)\right) \tag{17}$$

and then

$$\boxed{2\frac{\psi'-\psi''}{\psi'+\psi''} = \frac{1}{Q} = 2\frac{\arccos\left(1-\frac{1}{2}\frac{(1-|r|)^2}{|r|}\right)}{\varphi_r}} \tag{18}$$

This can be approximated for high value of |r| by

$$\boxed{\frac{1}{Q} \approx 2\frac{(1-|r|)}{\varphi_r\sqrt{|r|}}} \tag{19}$$

By the same way, we can evaluate Q_x, resolving the following equation :

$$\left|\frac{t}{1-r.\exp(-j\psi_x)}\right| = \frac{1}{\sqrt{x}}\sqrt{\frac{1+|r|}{1-|r|}}, \tag{20}$$

We find $$\boxed{\frac{1}{Q_x} = 2\frac{\arccos\left(1-\frac{1}{2}(x-1)\frac{(1-|r|)^2}{|r|}\right)}{\varphi_r}} \tag{21}$$

For high value of |r|, we can write :

$$\boxed{\frac{1}{Q_x} \approx \sqrt{x-1}\,\frac{1}{Q}}, \qquad \text{with} \qquad x = \frac{\left(|T_2(\psi)|_{max}{}^2\right)}{|T_2(\psi)|^2} \tag{22}$$